Chap 9- Inference in FOL and Reasoning

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Reasoning

- After knowledge representation, let's look at mechanisms to reasoning.
- Reasoning is the process of deriving logical conclusions from given facts.
- Durkin defines reasoning as ‘the process of working with knowledge, facts and problem solving strategies to draw conclusions’.
Types of Reasoning: Deductive

- **Deductive reasoning**: is based on deducing new information from logically related known information.

- A deductive argument offers assertions that lead automatically to a conclusion.

- **If there is dry wood, oxygen and a spark, there will be a fire.**

- Given: There is dry wood, oxygen and a spark
  
  We can deduce:
  
  - There will be a fire.
Types of Reasoning: Inductive

- **Inductive reasoning** is based on forming, or inducing a ‘generalization’ from a limited set of observations.

  **Observation**: All the crows that I have seen in my life are black.

  **Conclusion**: All crows are black.

- Thus the essential difference is that inductive reasoning is based on *experience* while deductive reasoning is based on *rules*, hence the latter will always be correct.
Types of Reasoning: Abductive

- Abductive reasoning: Deduction is exact in the sense that deductions follow in a logically provable way from the axioms.
- Abduction is a form of deduction that allows for probable inference, i.e. the conclusion might be wrong.
- Implication: She carries an umbrella if it is raining
  Axiom: she is carrying an umbrella
  Conclusion: It is raining
- This conclusion might be false, because there could be other reasons that she is carrying an umbrella e.g. she might be carrying it to protect herself from the sun.
Types of Reasoning: Analogical

- Analogical reasoning works by drawing analogies (similarities) between two situations, looking for similarities and differences.
- When you say driving a truck is just like driving a car, by analogy you know that there are some similarities in the driving mechanism.
- But you also know that there are certain other distinct characteristics of each.
Types of Reasoning: Common-sense

- Common-sense reasoning: Common-sense reasoning is an informal form of reasoning that uses rules gained through experience or what we call rules-of-thumb.
- It operates on heuristic (experimental) knowledge and heuristic rules.
Types of Reasoning: Non-Monotonic reasoning

- Non-Monotonic reasoning is used when the facts of the case are likely to change after some time.

- Rule:
  IF the wind blows
  THEN the curtains sway

- When the wind stops blowing, the curtains should sway no longer.

- However, if we use monotonic reasoning, this would not happen. The fact that the curtains are saying would be retained even after the wind stopped blowing.
Inference

- A process of deriving new information from known information.

- In the domain of AI, the component of the system that performs inference is called an inference engine.

- We will look at inference within the framework of ‘logic’, which we introduced earlier.

- We can use proof system:
  - Begin with initial premises of the proof (or knowledge base)
  - Use rules, i.e. apply rules to the known information
  - Add new statements, based on the rules that match
Inference....working example

**Knowledge Base**

- Rule 1:
  - IF father (X, Y)
  - AND father (X, Z)
  - THEN brother (Y, Z)

- Rule 2:
  - IF father (X, Y)
  - THEN payTuition (X, Y)

- Rule 3:
  - IF brother (X, Y)
  - THEN like (X, Y)

**Working Memory**

- father (M.Tariq, Ali)
- father (M.Tariq, Ahmed)

**Inference Engine**

- brother (? , ?)
- payTuition (? , ?)
- payTuition (? , ?)
- like (? , ?)
The table below gives the four rules of inference together:

<table>
<thead>
<tr>
<th>$\alpha \to \beta$</th>
<th>$\alpha \to \beta$</th>
<th>$\alpha$</th>
<th>$\alpha \land \beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>$\neg \beta$</td>
<td>$\beta$</td>
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<td>$\beta$</td>
<td>$\neg \alpha$</td>
<td>$\alpha \land \beta$</td>
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</tbody>
</table>

Modus Ponens      Modus Tolens  And-Introduction  And-elimination

Figure: Table of Rules of Inference
## Rules of Inference....An Example

<table>
<thead>
<tr>
<th>Step</th>
<th>Formula</th>
<th>Derivation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$A \land B$</td>
<td>Given</td>
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<tr>
<td>2</td>
<td>$A \rightarrow C$</td>
<td>Given</td>
</tr>
<tr>
<td>3</td>
<td>$(B \land C) \rightarrow D$</td>
<td>Given</td>
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<tr>
<td>4</td>
<td>$A$</td>
<td>1 And-elimination</td>
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<tr>
<td>5</td>
<td>$C$</td>
<td>4, 2 Modus Ponens</td>
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<tr>
<td>6</td>
<td>$B$</td>
<td>1 And-elimination</td>
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<tr>
<td>7</td>
<td>$B \land C$</td>
<td>5, 6 And-introduction</td>
</tr>
<tr>
<td>8</td>
<td>$D$</td>
<td>7, 3 Modus Ponens</td>
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</table>
Resolution Rule

<table>
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<tr>
<th>A</th>
<th>β</th>
<th>Γ</th>
<th>¬β</th>
<th>α ∨ β</th>
<th>¬β ∨ γ</th>
<th>α ∨ γ</th>
</tr>
</thead>
<tbody>
<tr>
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</table>

\[
\begin{align*}
\alpha \lor \beta \\
neg \beta \lor \gamma \\
\hline
\alpha \lor \gamma
\end{align*}
\]
Rules of Inference for Quantifiers

- **Universal Instantiation:**
  \[
  \forall x P(x) \quad \therefore P(c)
  \]

- **Universal Generalization:**
  
  \[
  P(c) \text{ for an arbitrary } c \quad \therefore \forall x P(x)
  \]

- **Existential Instantiation:**
  
  \[
  \exists x P(x) \quad \therefore P(c) \text{ for some element } c
  \]

- **Existential Generalization:**
  
  \[
  P(c) \text{ for some element } c \quad \therefore \exists x P(x)
  \]

\[
\forall x \ (H(x) \ \rightarrow \ M(x))
\]

\[
H(\text{Sachin})
\]

\[
\therefore M(\text{Sachin})
\]
Rules of Inference for Quantifiers

<table>
<thead>
<tr>
<th>Step</th>
<th>Valid Argument</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>$\forall x (H(x) \rightarrow M(x))$</td>
<td>Premise</td>
</tr>
<tr>
<td>(2)</td>
<td>$H(Sachin) \rightarrow M(Sachin)$</td>
<td>Universal instantiation from (1)</td>
</tr>
<tr>
<td>(3)</td>
<td>$H(Sachin)$</td>
<td>Premise</td>
</tr>
<tr>
<td>(4)</td>
<td>$M(Sachin)$</td>
<td>Modus ponens from (2) and (3)</td>
</tr>
</tbody>
</table>
Rules of Inference for Quantifiers

- “All lions are fierce.”
- “Some lions do not drink coffee.”
- Does it follow that: “Some fierce creatures do not drink coffee.”

1. \( \forall x \ (L(x) \rightarrow F(x)) \)
2. \( \exists x \ (L(x) \land \neg C(x)) \)
3. \( \exists x \ (F(x) \land \neg C(x)) \)
### Rules of Inference for Quantifiers

<table>
<thead>
<tr>
<th>Step</th>
<th>Predicate</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \exists x (L(x) \land \neg C(x)) )</td>
<td>Premise</td>
</tr>
<tr>
<td>2</td>
<td>( L(Foo) \land \neg C(Foo) )</td>
<td>Existential Instantiation from (1)</td>
</tr>
<tr>
<td>3</td>
<td>( L(Foo) )</td>
<td>Simplification from (2)</td>
</tr>
<tr>
<td>4</td>
<td>( \neg C(Foo) )</td>
<td>Simplification from (2)</td>
</tr>
<tr>
<td>5</td>
<td>( \forall x (L(x) \rightarrow F(x)) )</td>
<td>Premise</td>
</tr>
<tr>
<td>6</td>
<td>( L(Foo) \rightarrow F(Foo) )</td>
<td>Universal instantiation from (5)</td>
</tr>
<tr>
<td>7</td>
<td>( F(Foo) )</td>
<td>Modus ponens from (3) and (6)</td>
</tr>
<tr>
<td>8</td>
<td>( F(Foo) \land \neg C(Foo) )</td>
<td>Conjunction from (4) and (7)</td>
</tr>
<tr>
<td>9</td>
<td>( \exists x (F(x) \land \neg C(x)) )</td>
<td>Existential generalization from (8)</td>
</tr>
</tbody>
</table>
Am Inference rules that requires finding substitutions that make different logical expressions UNIFICATION look identical.

This process is called unification and is a key component of all first-order UNIFIER inference algorithms.

The UNIFY algorithm takes two sentences and returns a unifier for them if one exists:

\[ \text{UNIFY}(p, q) = \theta \text{ where } \text{SUBST}(\theta, p) = \text{SUBST}(\theta, q). \]
Suppose we have a query
AskVars(Knows(John, x)): whom does John know? Answers to this query can be found

UNIFY(Knows(John, x), Knows(John, Jane)) = {x/Jane}
UNIFY(Knows(John, x), Knows(y, Bill)) = {x/Bill, y/John}
UNIFY(Knows(John, x), Knows(y, Mother(y))) = {y/John, x/Mother(John)}
UNIFY(Knows(John, x), Knows(x, Elizabeth)) = fail.
UNIFY(Knows(John, x), Knows(x17, Elizabeth)) = {x/Elizabeth, x17/John}
Forward chaining

- Let’s look at how a doctor goes about diagnosing a patient. He asks the patient for symptoms and then infers diagnosis from symptoms.

- Forward chaining is based on the same idea. It is an “inference strategy that begins with a set of known facts, derives new facts using rules whose premises match the known facts, and continues this process until a goal state is reached or until no further rules have premises that match the known or derived facts”.

- It is a data-driven approach.
Forward chaining
Approach

- Add facts to working memory (WM)
- Take each rule in turn and check to see if any of its premises match the facts in the WM
- When matches found for all premises of a rule, place the conclusion of the rule in WM.
- Repeat this process until no more facts can be added. Each repetition of the process is called a pass.
An Example

Doctor example (forward chaining)

Rules

Rule 1
IF The patient has deep cough
AND We suspect an infection
THEN The patient has Pneumonia

Rule 2
IF The patient’s temperature is above 100
THEN Patient has fever

Rule 3
IF The patient has been sick for over a fortnight
AND The patient has a fever
THEN We suspect an infection

Case facts

- Patients temperature = 103
- Patient has been sick for over a month
- Patient has violent coughing fits
Backward chaining

- Backward chaining is an inference strategy that works backward from a hypothesis to a proof.
- You begin with a hypothesis about what the situation might be. Then you prove it using given facts.
- For Example a doctor may suspect some disease and proceed by inspection of symptoms.
- In backward chaining terminology, the hypothesis to prove is called the goal.
Backward chaining
Thanks ......

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